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David C. Stredulinsky



Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

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**DEFENCE RESEARCH ESTABLISHMENT ATLANTIC**

9 GROVE STREET

P.O. BOX 1012
DARTMOUTH, N.S.
B2Y 3Z7

TELEPHONE
(902) 426-3100

**CENTRE DE RECHERCHES POUR LA DÉFENSE ATLANTIQUE**

9 GROVE STREET

C.P. BOX 1012
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Approved by R.W. Graham:
Head/Hydronautics Section

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## Abstract

Ship structural design and analysis is moving away from empirical static design wave balance and towards more rational methods involving computer modelling of the sea load and structural response. A ship load history is usually only known or predicted in terms of an operational profile defined by the time spent in many combinations of different ship speeds, headings and sea environments. Each combination of speed, heading and the wave frequency spectrum defining the sea environment can be used to derive a frequency spectrum of the local structural response (such as stress, strain or stress intensity factor) at a location or region of the ship. The resulting response spectra can then be applied in a fatigue or ultimate strength assessment. This report proposes a method for the calculation of the frequency spectrum of the structural response based on the use of regular wave hull pressure loads and rigid body accelerations provided by PRECAL (a linear frequency domain hydrodynamics code based on 3D potential flow) and a top-down quasi-static structural analysis procedure to be implemented in the DND suite of finite element codes called VAST. Static finite element analyses are conducted for unit hydrodynamic facet pressure and rigid body acceleration load cases. This should reduce tremendously the computational effort required compared to directly applying a set of wave pressure loads for each combination of regular wave frequency, ship speed and heading needed to represent a ship operational profile. Transfer functions relating a regular unit amplitude wave directly to the structural response are calculated before computing the response spectra, eliminating the need to explicitly apply large cross spectral density matrices of hull pressure loads to the finite element model as is often done in classical random response methods.



## Résumé

Dans le domaine de l'analyse et de la conception des structures des navires, on s'éloigne aujourd'hui de la méthode empirique à conception statique d'équilibre sur les vagues pour passer à des méthodes plus rationnelles faisant appel à la modélisation sur ordinateur de la charge due aux vagues et de la réponse de la structure. Un historique de charge d'un navire n'est habituellement connu ou prévu que sous forme d'un profil opérationnel défini par le temps pendant lequel le navire a été soumis à un grand nombre de combinaisons de vitesses, de caps et de conditions ambiantes en mer. Chaque combinaison de vitesse, de cap et de spectre de fréquence des vagues définissant les conditions ambiantes en mer peut être utilisée pour déduire un spectre de fréquence de al réponse locale de la structure (par exemple contrainte, déformation ou facteur d'intensité de contrainte) en un point ou une partie du navire. Le spectre de réponse résultant peut ensuite être appliqué dans un essai de fatigue ou de résistance maximale. Le présent rapport propose une méthode de calcul du spectre de fréquence de la réponse de la structure basée sur l'utilisation de charges de pression exercées par des vagues normales sur la coque et d'accélérations de corps rigides fournies par PRECAL (un logiciel d'hydrodynamique du domaine de fréquence linéaire basé sur un régime d'écoulement potentiel tridimensionnel). Il propose aussi une méthode d'analyse des structures quasi statique descendante devant être mise en application dans la suite de programmes à éléments finis du MDN appelée VAST. Des analyses par éléments finis statiques sont effectuées pour des cas de charge avec des valeurs unitaires de pression de facette hydrodynamique et d'accélération de corps rigide. Cette façon de procéder devrait réduire considérablement la tâche de calcul nécessaire par comparaison à

l'application directe d'un ensemble de charges de pression des vagues pour chaque combinaison de fréquence de vagues normales et de vitesse et cap de navire requise pour représenter un profil opérationnel de navire. Des fonctions de transfert reliant directement une vague d'amplitude normale à la de réponse de la structure sont calculées avant la détermination des spectres de réponse, ce qui élimine la nécessité d'appliquer explicitement de grosses matrices de densités interspectrales de charges de pression exercées sur la coque au modèle à éléments finis, comme il faut souvent le faire dans les méthodes à réponse aléatoire classiques.

## Proposed Method for Calculation of Spectral Response to Random Wave Loading Using Top-Down Finite Element Modelling

by D.C. Stredulinsky

### Executive Summary

**Introduction:** Ship structural design and analysis is moving away from empirical static design wave balance and towards more rational methods involving computer modelling of the sea load and structural response. This is taking place in both naval and commercial sectors where tools such as the American Bureau of Shipping (ABS) SAFEHULL code for tankers are gaining popularity. The simplest methods consider wave loads in terms of vertical hull girder bending moments derived from empirical or 2D strip theory hydrodynamic models. More sophisticated models can include combined vertical and lateral bending moment load cases. The direct application of hull pressure loads is likely to give the most realistic predictions, accounting for both hull girder bending loads and local hull pressure loads. Hull pressure spectral loads can be computed from wave spectra using 3D hydrodynamic models and used in finite element analyses to determine the frequency spectra of the structural response. The response spectra can then be used to predict fatigue and ultimate strength performance. A ship operational profile typically involves a large number of combinations of wave environment, ship speed and ship headings, each of which produces a different hull pressure load case. The application of existing finite element based random response methods is impractical due to the large number of load cases which need be considered. DREA has proposed a new approach which is outlined in this report.

**Principal Results:** This report proposes a finite element based method in which loading is provided in terms of hull pressure frequency spectra, derived from wave spectra and ship operational conditions, to predict structural response spectra. The method is expected to considerably improve computational efficiency compared to classical random response methods now employed.

**Significance of Results:** The major development project, Improved Ship Structural Maintenance Management (ISSMM), and other CF hull system life cycle management initiatives, require the capability to predict realistic structural response of CF vessels to sea loads. The proposed method should make the prediction of realistic structural response frequency spectra and subsequent assessment of fatigue strength practical for realistic ship operational profiles. The method has the potential to give more realistic predictions of the structural response than simpler models involving only hull girder bending loads, since it includes both global loads and local wave loads.

**Future Plans:** The method will be implemented and tested as part of the initial phase of the ISSMM project by using the DND finite element code VAST and the 3D frequency domain hydrodynamics code PRECAL (used for linear sea loads prediction). If the method is found to give realistic predictions with an acceptable computational effort, it will be refined and integrated into ISSMM software. The extension of the method to handle nonlinear sea loads will also be undertaken.

# Contents

# 1 Introduction

Traditional ship structural analysis is based on a static equivalent beam analysis of the ship hull girder balanced on a design wave. The safety factor separating the applied load and the structural resistance has been derived over time through trial and error influenced by the sometimes opposing forces of concern for safety and economic efficiency. The result is an analysis process which works for design of conventional ship structures, but greatly oversimplifies a complex loading-response process and does not provide the rational means to assess through-life safety or develop more efficient designs.

Recent advances in computing technology are resulting in improved methods for modelling the loads acting on a ship operating in a defined seaway and the corresponding response of the complex ship structure. Both naval and commercial sectors are developing tools (such as the American Bureau of Shipping (ABS) SAFEHULL code for tankers) to bring rational ship structural analysis to reality. The major development project, Improved Ship Structural Maintenance Management (ISSMM) [1], and other CF hull system life cycle management initiatives, require the capability to predict realistic structural response to sea loads for CF vessels.

The Hydronautics Section at DREA has been developing methods for prediction of sea loads and their application to finite element models of the hull structure to predict fatigue and ultimate strength performance. Through cooperative research with the NSMB (Netherlands Ship Model Basin) Cooperative Research Ships organization, a linear three-dimensional seakeeping code, PRECAL [2], was developed to predict pressure spectra for the ship hull operating in a seaway. The PRECAL code has been used in conjunction with the DND finite element code VAST to predict stress spectra at critical details in the ship hull for a single ship speed and heading in a seaway defined by one wave energy spectrum [3] and the hull pressure predictions from PRECAL have been validated by DREA through full scale measurement [4]. The structural response in terms of stress spectra, strain spectra or stress intensity factor spectra can be used to predict fatigue crack initiation and crack growth behaviour for a given operating profile of a vessel. The PRECAL results can also be used with extremal theory to establish the most likely maximum linear loads on the hull structure to determine safety levels against ultimate strength.

A realistic ship operational profile involves a large number of combinations of sea environments, ship speeds and ship headings (defined as operational cells), each of which produces a different hull pressure spectral load case. Application of the existing random response methods developed in VAST [3, 5, 6] for a full ship finite element model and the large number of wave spectral load cases is impractical due to the large computational effort required to determine the structural response to all load cases. For crack propagation the finite element mesh must be changed after a certain increment of crack growth, and the structural response re-determined, further multiplying the computational effort required.

This report proposes a new method for calculation of the structural response frequency spectrum based on the use of hull pressure RAOs and rigid body acceleration RAOs provided by PRECAL and a top-down finite element structural analysis procedure. This report does not consider the subsequent application of the response spectra to fatigue or ultimate strength prediction.

The proposed method can be summarized as follows. The regular wave hull pressure transfer functions and rigid body acceleration transfer functions can be calculated with PRECAL for

load cases based on a matrix of ship speeds, heading angles and frequencies. The VAST finite element code is then used to derive quasi-static transfer functions from hydrodynamic mesh facet pressures and rigid body acceleration components to global finite element model nodal displacements. A local detailed finite element model is used to calculate the transfer function relating the hydrodynamic facet pressure loads and rigid body acceleration loads to the local response (stress, strain or stress intensity factors). The transfer functions can be combined to provide an overall transfer function between each regular wave and the local response which can be used in conjunction with the wave frequency spectrum to obtain the local response frequency spectrum. The method requires only one global finite element analysis per hydrodynamic facet. An alternative method, presently available in VAST, employs a finite element analysis for each hull regular wave load case (a combination of ship speed, heading angle and wave frequency) which even for linear analysis could require a much larger number of load cases compared to typically 200 facets used in a hydrodynamic mesh. This becomes even more critical for non-linear wave load cases where the number of load cases is equal to the number of linear load cases multiplied by the number of wave heights which need to be considered. The top-down procedure has the advantage that the global finite element analyses need only be done once and stored. Subsequent analyses then only involve local finite element computations. This should provide considerable time savings for crack propagation analysis which involves re-meshing of the area around the crack tip and repeated local model finite element calculations with every increment of crack growth. Three options for conducting the local analyses are outlined, two of which look feasible. By calculating the complete transfer function from the regular wave height to the local response before computing the response spectra, the requirement to apply a large matrix of cross spectral density of pressure loads to hull finite elements is also avoided. This is also likely to significantly improve the efficiency of the method over existing random response methods.

## 2 Classical Random Load Method

The classical equation for considering the forced response of a linear deterministic system subjected to stationary random loading is given by

$$[S_{\sigma\sigma}(\omega)] = [H_{\sigma F}(\omega)]\,[S_{FF}(\omega)]\,[H^*_{\sigma F}(\omega)]^T \tag{1}$$

where $[S_{\sigma\sigma}]$ is the cross spectral density of the structural response, for example the components of stress at a particular location on the structure, and $[S_{FF}]$ is the cross spectral density of applied random loads as a function of the forcing frequency $\omega$. The symbol $^*$ indicates the complex conjugate and $[X]^T$ indicates the transpose of the matrix $[X]$. $[H_{\sigma F}]$ is the deterministic transfer function between loads $\{f\}$ (harmonic with time $t$ and represented by $\{F\}\exp(i\omega t)$) and the harmonic response $\{\sigma\}\exp(i\omega t)$ defined by

$$\{\sigma(\omega)\} = [H_{\sigma F}(\omega)]\,\{F(\omega)\} \tag{2}$$

where $\{\,\}$ represents a column vector and $i$ represents $\sqrt{-1}$. The elements of $\{\sigma\}$, $\{F\}$ and $[H_{\sigma F}]$ are complex numbers which account for phase differences between the harmonic quantities. Each harmonic quantity, for example the force component $f_j$, may be specified in terms of

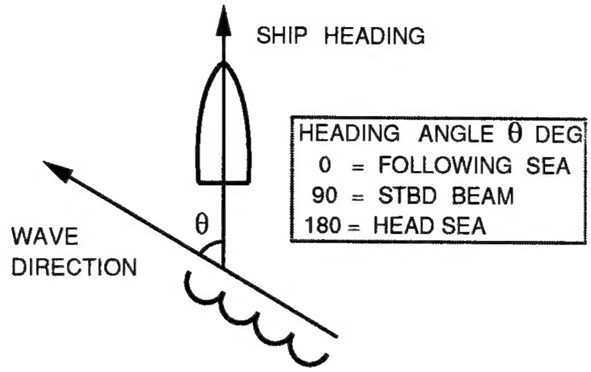



Figure 1: Convention for the the relative heading angle between wave direction and ship heading used in PRECAL

an amplitude $\hat{f}_j$ and phase angle $\alpha_j$. In this case the complex force amplitude $F_j$ used in Equation 2 is given by

$$F_j = \hat{f}_j \cos(\alpha_j) + i\hat{f}_j \sin(\alpha_j). \tag{3}$$

A procedure has been developed in the VAST finite element code [5, 6] using the modal frequency response method or direct frequency response method to determine the response cross spectral density matrix $[S_{\sigma\sigma}]$ for a given cross spectral matrix of applied nodal forces $[S_{FF}]$. A method has also been developed to calculate this cross spectral density matrix of nodal forces $[S_{FF}]$ for a given directional wave spectrum $S_{\eta\eta}$ and ship speed $V$ based on the equation

$$[S_{FF}(\omega,\theta)] = \{H_{F\eta}(\omega,\theta)\}\, S_{\eta\eta}(\omega,\theta) \left\{H^*_{F\eta}(\omega,\theta)\right\}^T \tag{4}$$

where the transfer function vector $\{H_{F\eta}\}$ between the nodal forces $\{F\}$ and a regular wave of amplitude $\eta$ is calculated using PRECAL and algorithms to transfer between PRECAL hydrodynamic mesh hull pressures and the finite element mesh nodal forces. Since a directional wave spectrum is considered, the matrices shown in Equation 4 are considered as function of the wave frequency $\omega$ and the relative heading angle $\theta$ between the wave direction and ship heading. The relative heading angle can be defined as shown in Figure 1 based on the convention used in PRECAL. The resulting response spectral density $[S_{\sigma\sigma}]$ from Equation 1 also becomes a function of wave frequency and heading angle. For a given ship speed, heading angle and wave frequency, a dynamic analysis can be conducted at the encounter frequency $\omega_e$, the forcing frequency seen by the structure, to determine the structural response.

When applied to a full ship finite element model, the above method is computationally intensive. Also the entire finite element analysis would have to be repeated for each ship speed, heading angle and wave spectra combination (potentially 500 to 1000 load cases for linear wave loading and possibly combined with several wave heights if nonlinear sea loads are considered resulting in several thousand cell load cases). In a top-down modelling scenario it would be desirable to compute and store the global model displacement response spectra $S_{UU}(\omega)$ for each cell. Using the classical random response method the cross spectral density of displacement would have to be stored for each of the degrees of freedom (DOF) in the global

model for each cell load case. If the global model contained 10000 DOF this would require storage of $10^8$ spectra $S_{UU}(\omega)$ for each load case. Clearly the method is likely to be impractical based on the computation time requirements and disk storage requirements.

Since most of the wave spectral energy is usually at frequencies well below the lowest hull vibration mode, the low frequency response can be considered in a quasi-static analysis outlined in the following sections to calculate transfer function $[H_{\sigma F}]$ between the hull loads and structural response. Since the cross spectral density matrix of loads is derived from a single wave spectrum it is also not necessary to use this load matrix explicitly. Combining Equation 1 and Equation 4 gives

$$
\begin{aligned}
[S_{\sigma\sigma}] &= [H_{\sigma F}]\{H_{F\eta}\} S_{\eta\eta} \left\{H^*_{F\eta}\right\}^T [H^*_{\sigma F}]^T & (5)\\
&= ([H_{\sigma F}]\{H_{F\eta}\}) S_{\eta\eta} ([H_{\sigma F}]\{H_{F\eta}\})^{*T} \\
&= \{H_{\sigma\eta}\} S_{\eta\eta} \left\{H^*_{\sigma\eta}\right\}^T . & (6)
\end{aligned}
$$

Thus if the transfer function $\{H_{\sigma\eta}\}$ from regular wave to the structural response is calculated from

$$\{H_{\sigma\eta}\} = [H_{\sigma F}]\{H_{F\eta}\} \tag{7}$$

then the cross spectral density of the response $[S_{\sigma\sigma}]$ can be obtained using Equation 6 above without calculating the cross spectral density of loads matrix $[S_{FF}]$ explicitly. It is implicitly included by the middle three terms in Equation 5.

# 3 Global Model Analysis

In the proposed method a top-down modelling procedure is employed involving a global coarse finite element model of the entire ship and a local detailed finite element model of the area of interest. The global model is used to calculate global displacements which are subsequently used in the local analysis. This section will consider analysis of the global model only. In the previous section the transfer function from wave to structural response $\{H_{\sigma\eta}\}$ was calculated based on Equation 7 which requires the transfer function $[H_{\sigma F}]$ between hull nodal forces (or element pressures) and the structural response. Each column of $[H_{\sigma F}]$ represents the global response calculated from a finite element analysis with a unit force applied to one of the wetted hull nodes (or alternatively with a unit pressure applied to one of the finite elements on the wetted surface of the hull). Since the number of hull elements is smaller than the number of nodes, using pressure loads on each element would result in fewer finite element runs required to determine $[H_{\sigma F}]$ than if nodal forces were used. Since the number of facets in the PRECAL hydrodynamic mesh is likely to be significantly lower than the number of wetted hull structural finite elements, further reduction can be obtained by formulating the transfer function between the regular wave and the structural response as

$$\{H_{U\eta}\} = [H_{UP}]\{H_{P\eta}\} \tag{8}$$

where $[H_{P\eta}]$ is the transfer function between the regular wave and the hydrodynamic mesh facet pressures, which can be computed by PRECAL, and $[H_{UP}]$ is the transfer function between the hydrodynamic mesh facet pressures and the structural response which in this case is taken to be

the displacements $\{U\}$ at all nodal DOF in the global model. Each column of $[H_{UP}]$ represents the complex displacement amplitudes calculated from the dynamic analysis of the global finite element model with a unit amplitude harmonic pressure applied over the finite elements of the hull corresponding to one of the hydrodynamic mesh facets and must be calculated at each encounter frequency in the input wave spectrum. This still involves significant finite element computational effort and when forcing frequencies are well below the first natural frequency of vibration of the structure, as is generally the case for wave loading, a quasi-static approach can be employed.

In the quasi static approach the real and imaginary components of the hull pressure loads and acceleration forces due to rigid body motion are applied to the global finite element model and the real and imaginary components of the resulting deformation displacements are calculated from static finite element analyses. PRECAL can be used to provide the amplitudes and phase angles for both the hydrodynamic mesh facet pressures $\{P\}$ and the six components of rigid body acceleration $\{A\}$ (surge, sway, heave, roll, pitch, or yaw) of the ship center of gravity (CG) for a unit amplitude regular wave at a given wave frequency, heading angle and ship speed. Conversion of the amplitudes and phase angles to complex amplitudes yields the transfer function from wave height to hydrodynamic facet pressures and rigid body accelerations $\left\{H_{\frac{A}{P}\eta}\right\}$. For a given wave amplitude $\eta$, the complex facet pressures and acceleration of the ship CG are then given by

$$\left\{ \begin{array}{c} \{A\} \\ \{P\} \end{array} \right\} = \left\{H_{\frac{A}{P}\eta}\right\}\eta. \tag{9}$$

The wave to global model nodal displacement transfer function can then be defined as

$$\{H_{U\eta}\} = \left[H_{U\frac{A}{P}}\right]\left\{H_{\frac{A}{P}\eta}\right\} \tag{10}$$

where $\left[H_{U\frac{A}{P}}\right]$ is the transfer function relating the hydrodynamic facet pressures and CG rigid body accelerations to the displacements of the nodes of the global finite element model. Each column of $\left[H_{U\frac{A}{P}}\right]$ can be determined by running a static finite element analysis with either a unit pressure over one hydrodynamic facet or a unit rigid body acceleration load (a translational acceleration component or a rotational acceleration component about the CG). The number of facets in the PRECAL hydrodynamic mesh is likely to be on the order of 200. This method would require, in the case of 200 hydrodynamic facets, running quasi-static finite element analyses for 206 finite element load cases and storing the nodal displacements for each case. Typical load cases are given in Figure 2 including one hydrodynamic facet unit pressure case, a rigid body translational acceleration in the vertical direction and a rigid body rotational acceleration about the vertical axis. Six finite element nodal displacement DOF have been constrained to eliminate rigid body modes.

The application of translational acceleration loads is relatively straight forward. A rigid body acceleration in the $x$ direction $a_x$, for example, would result in inertial static loads of $-m_i a_x$ applied at node $i$, for $i = 1$ to $N_S$ (the number of nodes in the global model) where $m_i$ is the nodal mass. VAST incorporates an option to specify the acceleration component and will automatically generate the translational inertial loads required. The application of rotational accelerations is not as straight forward. Assume a harmonic rigid body rotation about one of the global axes, for example $\phi_x$ about the $x$ axis, given by $\phi_x(t) = \Phi_x \cos(\omega_e t)$. Also consider

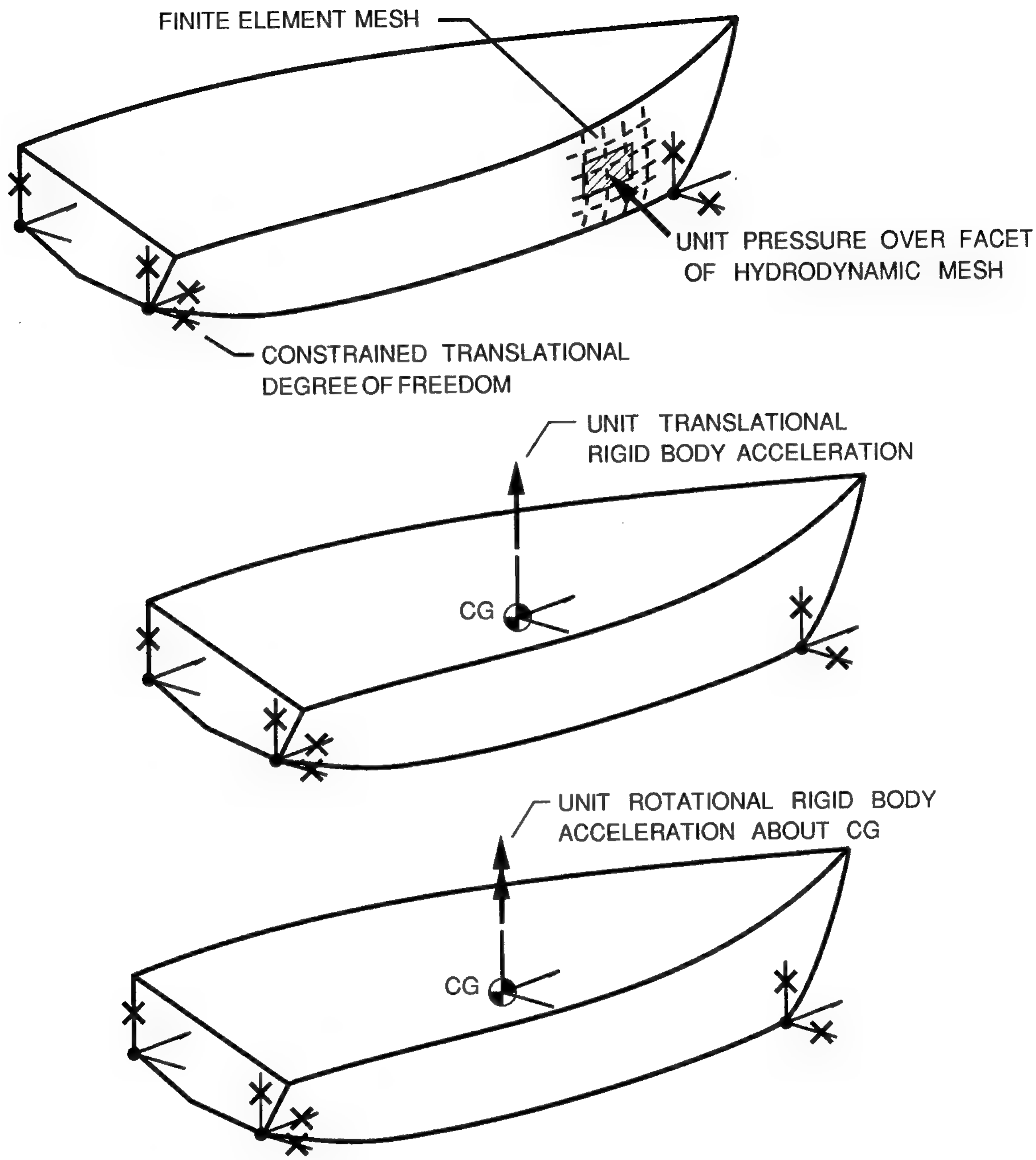



Figure 2: Example unit facet pressure and unit acceleration load cases for quasi-static global analyses

the plane perpendicular to the axis of rotation through node $i$ (at rectangular coordinates $x_i$,$y_i$,$z_i$) and the projection of the line between the CG and node $i$ in this plane. The length of the projected line will be called $R_p$ (see Figure 3). Application of the rotational acceleration about the CG results in a rotational acceleration at node $i$ of the same value and translational acceleration components in the plane perpendicular to the axis of rotation, a radial acceleration $a_r$ along the projection line and a tangential component $a_t$ perpendicular to the projection line. If the amplitude $\Phi_x$ of the rotation is small compared to one radian then

$$\begin{aligned} a_t(t) &= -R_p \Phi_x \omega_e^2 \cos(\omega_e t) = R_p \ddot{\phi}_x \\ &= R_p A_x \cos(\omega_e t) \qquad (11) \\ a_r(t) &= R_p \Phi_x^2 \omega_e^2 \cos(2\omega_e t) \\ &= -R_p \Phi_x A_x \cos(2\omega_e t) \qquad (12) \end{aligned}$$

where $\ddot{\phi}_x$ is the angular acceleration about the $x$ axis through the CG with amplitude $A_x$. The radial centrifugal component is a non-linear component proportional to the square of the rotational amplitude and occurs at twice the frequency of the CG angular rotation. Comparing equations 11 and 12 shows that the ratio of radial acceleration amplitude to the tangential acceleration amplitude $A_r/A_t$ is equal to the amplitude of the angle of rotation $\Phi_x$ in radians. Given a rotational amplitude of 5 degrees the radial acceleration component amplitude is 9 percent of the tangential component. The tangential component is easily incorporated into the quasi-static approach since VAST includes an option to supply a constant rotational acceleration about a specified point and calculate the resulting structural deformation and stresses. The non-linear radial component resulting from the harmonic rotational accelerations about the ship CG supplied by PRECAL will not be included in the method, although it is acknowledged that it could be significant compared to the tangential component in severe seas, especially in conditions giving large roll angles.

As previously indicated, to run a quasi-static finite element analysis, the model must be constrained from rigid body motion which requires constraint of six displacement DOF. If the hull pressure and inertial acceleration loads balance, as ideally they should, then the reaction forces at the constrained DOF should be zero. Differences, between the discretization mass and how loads are applied in the structural finite element model compared to the hydrodynamic model, may lead to some unbalance of quasi-static forces. The balance of forces can be checked or adjusted with the following method. During running of the VAST static analyses with unit load cases, the six reaction forces at constrained DOF can be used to form the transfer functions $\left[H_{R^A_P}\right]$ relating rigid body accelerations and facet pressures to the six reaction forces at the constrained DOF. These can be combined with any PRECAL wave pressure and acceleration load case to give the wave to reaction transfer function:

$$\{H_{R\eta}\} = \left[H_{R^A_P}\right] \left\{H_{^A_P\eta}\right\}. \qquad (13)$$

Thus for any PRECAL load case, the reaction forces $\{H_{R\eta}\}$ can be checked to see if they are close to zero.

If the reactions are assumed to be zero, then it is possible to calculate the required accelerations for a given set of PRECAL facet pressures. This can be done by partitioning $\left[H_{R^A_P}\right]$ between reactions due to unit acceleration loads and reactions due to unit facet pressure loads

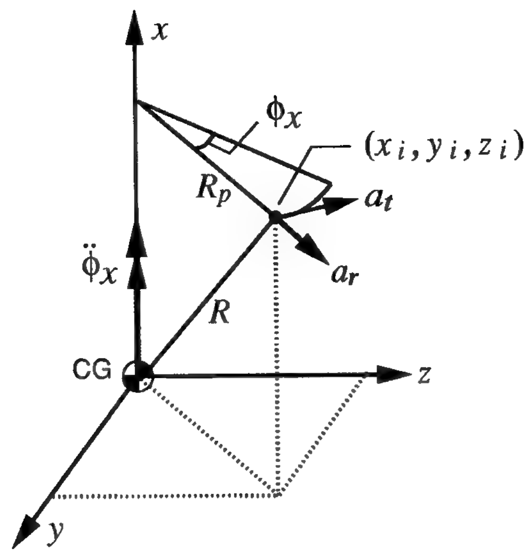



Figure 3: Translational accelerations $a_t$ and $a_r$ at point $(x_i, y_i, z_i)$ due to a rigid body rotational acceleration about the $x$ axis through the ship CG

as

$$\left[H_{R^A_P}\right] = \left[\begin{array}{cc} H_{RA} & H_{RP} \end{array}\right] \tag{14}$$

and partitioning the facet pressures and CG accelerations due to a unit amplitude wave as

$$\left\{H_{^A_P\eta}\right\} = \left\{\begin{array}{c} H_{A\eta} \\ H_{P\eta} \end{array}\right\}. \tag{15}$$

Setting the reactions to zero (the LHS of Equation 13) then gives the following equation for the required rigid body accelerations,

$$\{H_{A\eta}\} = -\left[H_{RA}\right]^{-1}\left[H_{RP}\right]\{H_{P\eta}\}. \tag{16}$$

This can be checked against the original PRECAL rigid body accelerations for each load case. In theory the location of the six constrained DOF is not critical as long as only rigid body modes are constrained and there is no constraint of deformation of the structure. Some unbalance could likely be tolerated as long as the resulting reaction forces are not too large and are not close to areas of the model where the local analysis is to be undertaken. Some calculations will be carried out to determine the degree of unbalance likely to occur based on typical hydrodynamic and structural models and to quantify the effect of the unbalance on structural response predictions.

In summary, prior to conducting any local response analysis, separate global static finite element analyses would be conducted with either a unit pressure load on each of the hydrodynamic mesh facets or a unit rigid body acceleration load. The resulting displacements at all global model DOF would form, for each load case, a column of the transfer matrix $\left[H_{U^A_P}\right]$ (real elements) relating applied facet pressures $\{P\}$ and rigid body acceleration components $\{A\}$ to the global displacements $\{U\}$. The transfer matrix $\left[H_{U^A_P}\right]$ would be calculated once

and stored. At the same time transfer functions $\left[H_{R\frac{A}{P}}\right]$ relating the applied facet pressures and rigid body acceleration components to the six reaction forces providing rigid body motion constraints would be calculated and stored. A flow chart for the global analysis is shown in Figure 4.

Also prior to conducting the local response analysis PRECAL would be run for each load case (a combination of wave frequency $\omega_i$, heading angle $\theta_j$ and ship speed $V_k$), likely to occur in the cells of wave spectra, ship heading and speed forming the ship operational profile, to determine the the transfer functions $\left\{H_{\frac{A}{P}\eta}\right\}_{ijk}$ relating the hydrodynamic pressures and ship CG rigid body accelerations to unit amplitude regular waves. $\left\{H_{\frac{A}{P}\eta}\right\}_{ijk}$ would be stored for each $\omega_i$, $\theta_j$, $V_k$ combination as shown in the flow chart for the PRECAL analysis given in Figure 5.

Once the PRECAL and global finite element runs are completed and the information stored then any number of local analyses could be conducted at any location in the ship without having to repeat the global finite element analyses or PRECAL analyses.

## 4 Local Finite Element Analysis

A local finite element analysis will be used to determine the stress or strain spectra at a location (or locations) for a given cell of wave spectrum, ship speed and heading which can be used in a fatigue crack initiation calculation or to determine the stress intensity factor spectra to be used in the calculation of an increment of crack growth. When the crack increment reaches a length which is likely to cause a significant change in the stress intensity factor, then the local model will have to be re-meshed and the local finite element analysis repeated. The following discussion will consider the local response in terms of the stress components $\{\sigma\}$ but will also apply to strain $\{\epsilon\}$ or stress intensity factor components $\{K\}$. The nodes on the boundary of the local model common to the global model will be considered as master nodes where global displacement boundary constraints will be applied to the local model. All other nodes of the local model on the common boundary but which are not in the global model will be treated as 'slave' nodes and constraint equations used to define the slave node displacements in terms of master node displacements.

The overall approach involves determining transfer functions $\{H_{\sigma\eta}\}_{ijk}$ between wave amplitude and the local stress components for all combinations of wave frequency $\omega_i$, heading angle $\theta_j$ and ship speed $V_k$ that are in the cells forming the ship operational profile. The directional wave spectral density $S_{\eta\eta}(\omega,\theta)$ will be defined discretely as $S_{\eta\eta}^{ij}$ over a number of wave frequencies $\omega_i$ and headings $\theta_j$ for a cell of the ship operation at speed $V_k$. The corresponding cross spectral density of local response $[S_{\sigma\sigma}]_{ij}$ is then given by

$$[S_{\sigma\sigma}]_{ij} = \{H_{\sigma\eta}\}_{ijk} S_{\eta\eta}^{ij} \left\{H_{\sigma\eta}^*\right\}_{ijk}^T. \tag{17}$$

There are at least three approaches that can be taken to compute the transfer function component $\{H_{\sigma\eta}\}_{ijk}$, the choice of which can be determined initially based on the number of finite element load cases which have to be considered in the analysis. The approaches are discussed in the following subsections of the report.

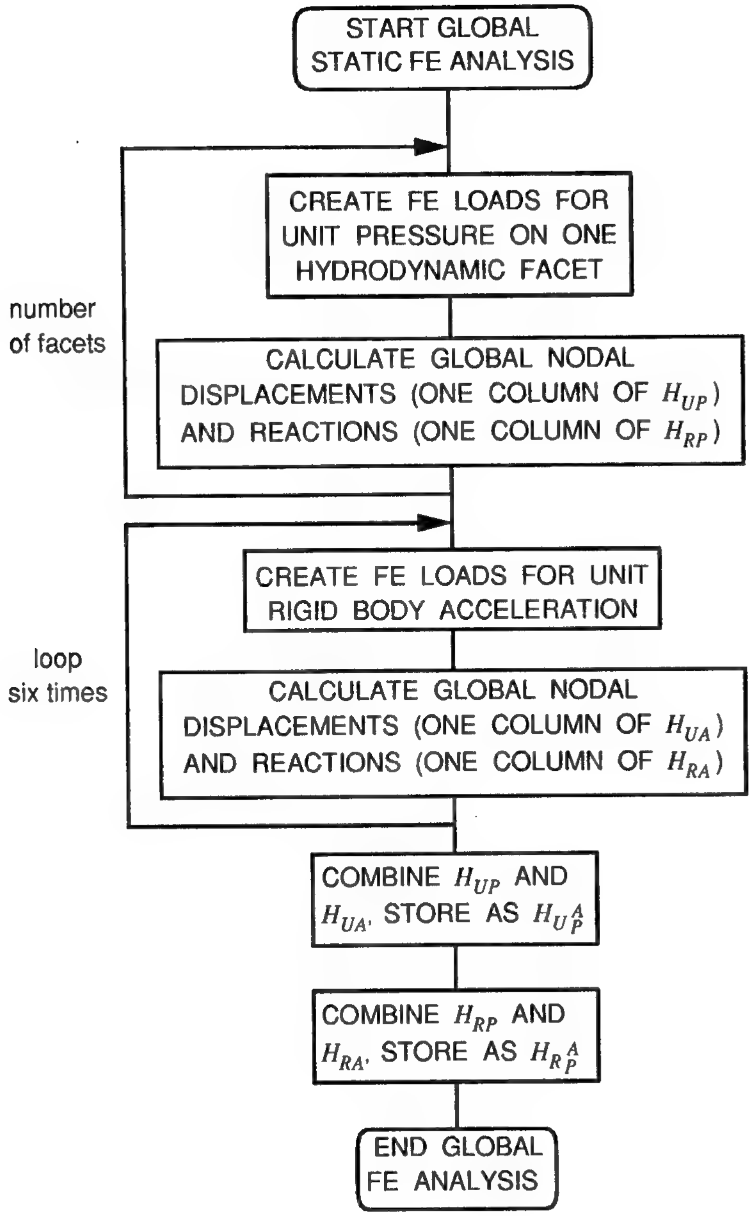



Figure 4: Flow chart of the global finite element analysis to produce nodal displacement and reaction force transfer functions

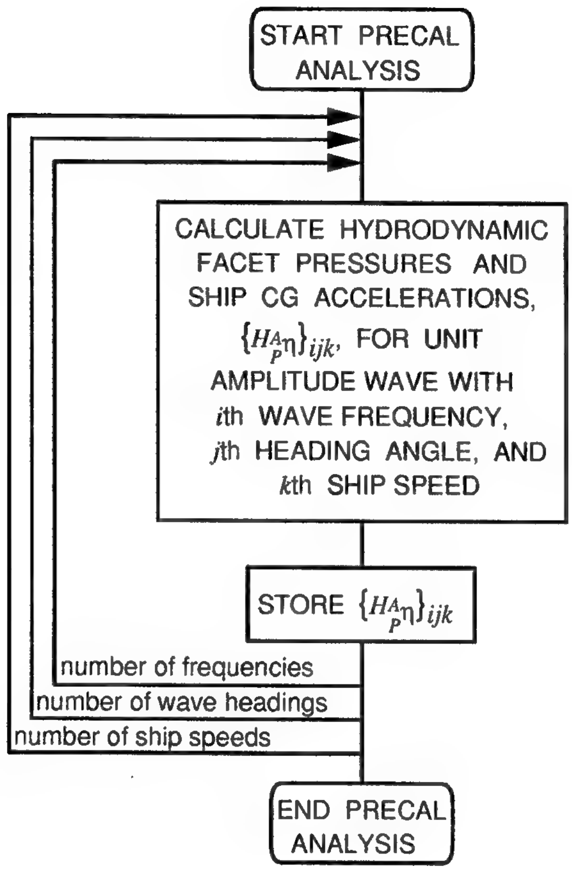



Figure 5: Flow chart of the PRECAL analysis to produce hull facet pressures and rigid body acceleration transfer functions

## 4.1 Direct Method

For a given wave frequency $\omega_i$, heading angle $\theta_j$ and ship speed $V_k$ the associated transfer function $\left[H_{\frac{A}{P}\eta}\right]_{ijk}$ between the wave amplitude and the hydrodynamic facet pressure and rigid body accelerations can be recalled and combined with the stored global model transfer function $\left[H_{U\frac{A}{P}}\right]$. For this load case the local master node displacements $\{u\}_{ijk}$ for a unit amplitude wave are then given by

$$\{u\}_{ijk} = [H_{uU}] \left[H_{U\frac{A}{P}}\right] \left\{H_{\frac{A}{P}\eta}\right\}_{ijk} \tag{18}$$

where $[H_{uU}]$ is a matrix of 0's and 1's which extracts from $\left[H_{U\frac{A}{P}}\right]$ the rows corresponding to only the global nodes which are master nodes in the local model. For computational purposes Equation 18 can be computed as

$$\{u\}_{ijk} = \left[H_{u\frac{A}{P}}\right] \left\{H_{\frac{A}{P}\eta}\right\}_{ijk} \tag{19}$$

where $\left[H_{u\frac{A}{P}}\right]$ are the rows of $\left[H_{U\frac{A}{P}}\right]$ corresponding to the local model boundary master node displacements. Two finite element analyses would then be conducted, one for the real parts and one for the imaginary parts of the boundary displacements $\{u\}_{ijk}$. In addition to the boundary constraints $\{u\}$ acting on the local finite element model, rigid body translational and rotational acceleration components extracted from $\left\{H_{\frac{A}{P}\eta}\right\}_{ijk}$ would be simultaneously applied in the analysis and if the local model included wetted hull surfaces any hydrodynamic facet pressure loads acting on the local model would also be extracted from $\left\{H_{\frac{A}{P}\eta}\right\}_{ijk}$ and simultaneously applied to the model. The resulting stress components from the finite element analyses would form directly the real and imaginary elements of the transfer function $\{H_{\sigma\eta}\}_{ijk}$ between wave amplitude and the local stress components which can then be used in Equation 17 to calculate the cell stress spectral density $[S_{\sigma\sigma}]_{ij}$ for each discrete wave frequency $\omega_i$ and heading angle $\theta_j$ combination associated with the cell wave spectra.

## 4.2 Local Unit Load Method

In this approach, the transfer function $\{H_{\sigma\eta}\}_{ijk}$ is calculated using the following equation

$$\{H_{\sigma\eta}\}_{ijk} = \left[H_{\sigma\frac{A}{P}}\right] \left\{H_{\frac{A}{P}\eta}\right\}_{ijk} \tag{20}$$

where $\left[H_{\sigma\frac{A}{P}}\right]$ is the transfer function relating the hydrodynamic facet pressures and rigid body accelerations to the local response. This transfer function can be split into two components as

$$\left[H_{\sigma\frac{A}{P}}\right] = [H_{\sigma l}] \left[H_{l\frac{A}{P}}\right] \tag{21}$$

where $[H_{\sigma l}]$ is the transfer function relating the local 'loads' $l$, which include master node applied boundary displacements $\{u\}$, local translational and rotational accelerations $\{a\}$ due to ship CG rigid body accelerations $\{A\}$, and local facet pressure loads $\{p\}$ (if the local model is on the wetted hull surface), to the local response $\{\sigma\}$. The transfer function $\left[H_{l\frac{A}{P}}\right]$ relates the applied global facet pressure and acceleration loads to the local loads. $[H_{\sigma l}]$ can be determined

by applying a number of unit load cases to the local model and calculating the local response and is defined according to the equation

$$\{\sigma\} = [H_{\sigma l}] \left\{ \begin{array}{c} \{a\} \\ \{p\} \\ \{u\} \end{array} \right\}. \tag{22}$$

The load cases include setting each of the master node displacement DOF to unity while setting all other master node displacements and local acceleration and pressures to zero (Figure 6a). Six unit acceleration load cases (holding master node displacements to zero) must also be considered (Figure 6b and 6c). The local accelerations $\{a\}$ can be set equal to the global accelerations $\{A\}$ if the local unit rotational acceleration load cases are applied with respect to the ship CG as can be specified in VAST. If the local model contains part of the wetted hull surface then load cases of unit pressures over each hydrodynamic facet area intersecting the local model hull area (setting other facet pressures and the local model boundary master node displacements to zero) would also have to be considered (Figure 6d). The resulting local finite element model response for each load case would form a column of the transfer matrix $[H_{\sigma l}]$

The local model loads can be related to facet pressure and CG accelerations by the equations

$$\{a\} = [H_{aA}]\{A\} \tag{23}$$

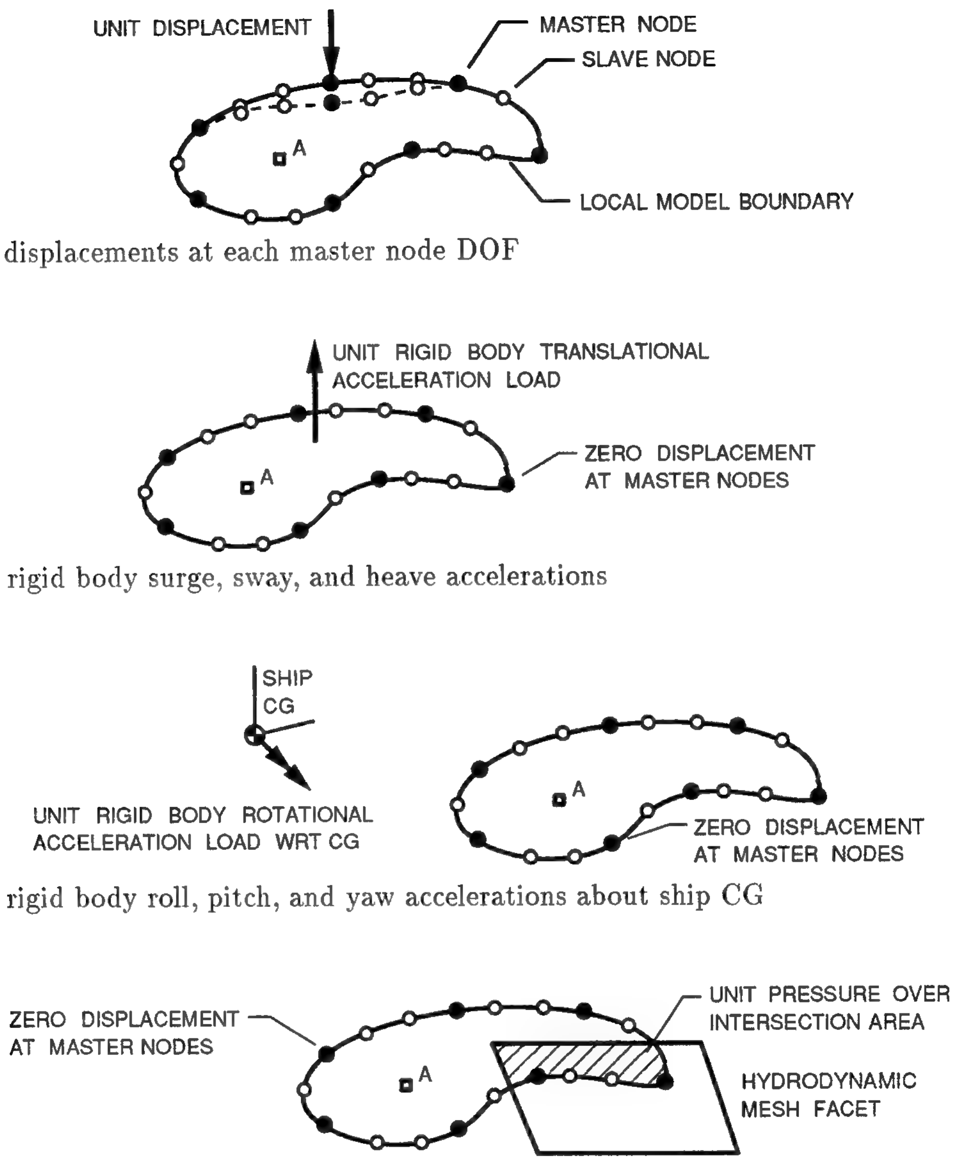
$$\begin{aligned} \{u\} &= [H_{uU}] \left[H_{U\frac{A}{P}}\right] \left\{ \begin{array}{c} \{A\} \\ \{P\} \end{array} \right\} \\ &= [H_{uU}] \left[ \begin{array}{cc} H_{UA} & H_{UP} \end{array} \right] \left\{ \begin{array}{c} \{A\} \\ \{P\} \end{array} \right\} \end{aligned} \tag{24}$$

$$\{p\} = [H_{pP}]\{P\}. \tag{25}$$

The matrix $[H_{uU}]$ was defined in Section 4.1 which when multiplied by the matrix $\left[H_{U\frac{A}{P}}\right]$ forms a matrix $\left[H_{u\frac{A}{P}}\right]$ which contains only the rows of $\left[H_{U\frac{A}{P}}\right]$ associated with displacements at the boundary master nodes for the particular local model under consideration. In the computer implementation, $\left[H_{u\frac{A}{P}}\right]$ is likely to be formed directly by extracting the appropriate rows from the entire global model transfer function $\left[H_{U\frac{A}{P}}\right]$. In Equation 24, $\left[H_{U\frac{A}{P}}\right]$ is also partitioned between columns associated with global rigid body accelerations $[H_{UA}]$ and columns associated with facet pressure loads $[H_{UP}]$. $[H_{aA}]$ relates the CG rigid body accelerations to the local model rigid body accelerations. Ignoring the nonlinear centrifugal acceleration and as indicated above if the local accelerations are applied in VAST about the ship CG then $[H_{aA}]$ becomes a 6 element by 6 element identity matrix $[I]$. The matrix $[H_{pP}]$ contains zeros and ones and extracts the local facet pressures from the vector of all the global model facet pressures.

The overall transfer matrix $\left[H_{l\frac{A}{P}}\right]$ needed to form $\left[H_{\sigma\frac{A}{P}}\right]$ in Equation 21 relates the facet pressures to local model loads as follows

$$\left\{ \begin{array}{c} \{a\} \\ \{p\} \\ \{u\} \end{array} \right\} = \left[H_{l\frac{A}{P}}\right] \left\{ \begin{array}{c} \{A\} \\ \{P\} \end{array} \right\} \tag{26}$$




a) Unit displacements at each master node DOF

b) Unit rigid body surge, sway, and heave accelerations

c) Unit rigid body roll, pitch, and yaw accelerations about ship CG

d) Unit pressures for any hydrodynamic mesh facets intersecting the local model

Figure 6: Example unit load cases for quasi-static local analyses with the Local Unit Load Method to determine structural response at internal node $A$

Based on the above equation and equations 23, 24, and 25 it can be shown that the transfer matrix relating facet pressures and global accelerations to the local model loads and boundary displacements is given by

$$\left[H_{l^A_P}\right] = \begin{bmatrix} [I] & [0] \\ [0] & [H_{pP}] \\ [H_{uU}]\,[H_{UA}] & [H_{uU}]\,[H_{UP}] \end{bmatrix} \tag{27}$$

where [0] is a matrix of zeros filling the empty sections of the partitions.

For computational purposes $\left[H_{l^A_P}\right]$ given by Equation 27 can be substituted into equation 21 and expanded in terms of partitioned quantities to give

$$\left[H_{\sigma^A_P}\right] = \left[\ [H_{\sigma A}] + [H_{\sigma u}]\,[H_{uA}] \quad [H_{\sigma p}] + [H_{\sigma u}]\,[H_{uP}]\ \right] \tag{28}$$

where the transfer matrix $[H_{\sigma l}]$ relating local response to local loads has been partitioned between columns corresponding to acceleration loads, pressure loads, and boundary displacements as

$$[H_{\sigma l}] = \left[\ H_{\sigma A} \quad H_{\sigma p} \quad H_{\sigma u}\ \right]. \tag{29}$$

The transfer function $\left[H_{\sigma^A_P}\right]$ can then be used to calculate the transfer function $\{H_{\sigma\eta}\}_{ijk}$ for a given PRECAL load case using Equation 20. A flow chart for the local unit load method is shown in Figure 7.

## 4.3 Unit Facet Pressure Method

In the global model analysis the transfer function $\left[H_{U^A_P}\right]$ will be stored. Each column of this matrix represents the global model nodal displacements for either a unit acceleration component applied at the ship CG or a unit pressure applied over a hydrodynamic mesh facet. Extraction of the rows of $\left[H_{U^A_P}\right]$ corresponding to the local model boundary master node displacements $\{u\}$ gives the matrix $\left[H_{u^A_P}\right]$. The transfer function $\left[H_{\sigma^A_P}\right]$ relating the facet pressure and CG accelerations to the local response can be obtained by applying in turn each column of $\left[H_{u^A_P}\right]$ as boundary displacements on the local model in conjunction with the unit CG acceleration if the column is associated with a global unit acceleration load or a unit facet pressure if the column is associated with a facet pressure and that facet is completely or partially contained in the local model. The local model load cases are illustrated in Figure 8 and a flow chart of the method is shown in Figure 9. The resultant structural response for each load case forms a column of the transfer function $\left[H_{\sigma^A_P}\right]$ which can be used to calculate the transfer function $\{H_{\sigma\eta}\}_{ijk}$ for a given PRECAL load case using Equation 20 as in the local unit load method.

## 4.4 Comparison of Local Analysis Methods

The efficiency of the three methods will depend mainly on the number of local finite element static load cases that must be employed in each method in the process of obtaining $\{H_{\sigma\eta}\}_{ijk}$ for all the PRECAL load cases (combinations of discrete wave frequency $\omega_i$, heading angle $\theta_j$ and

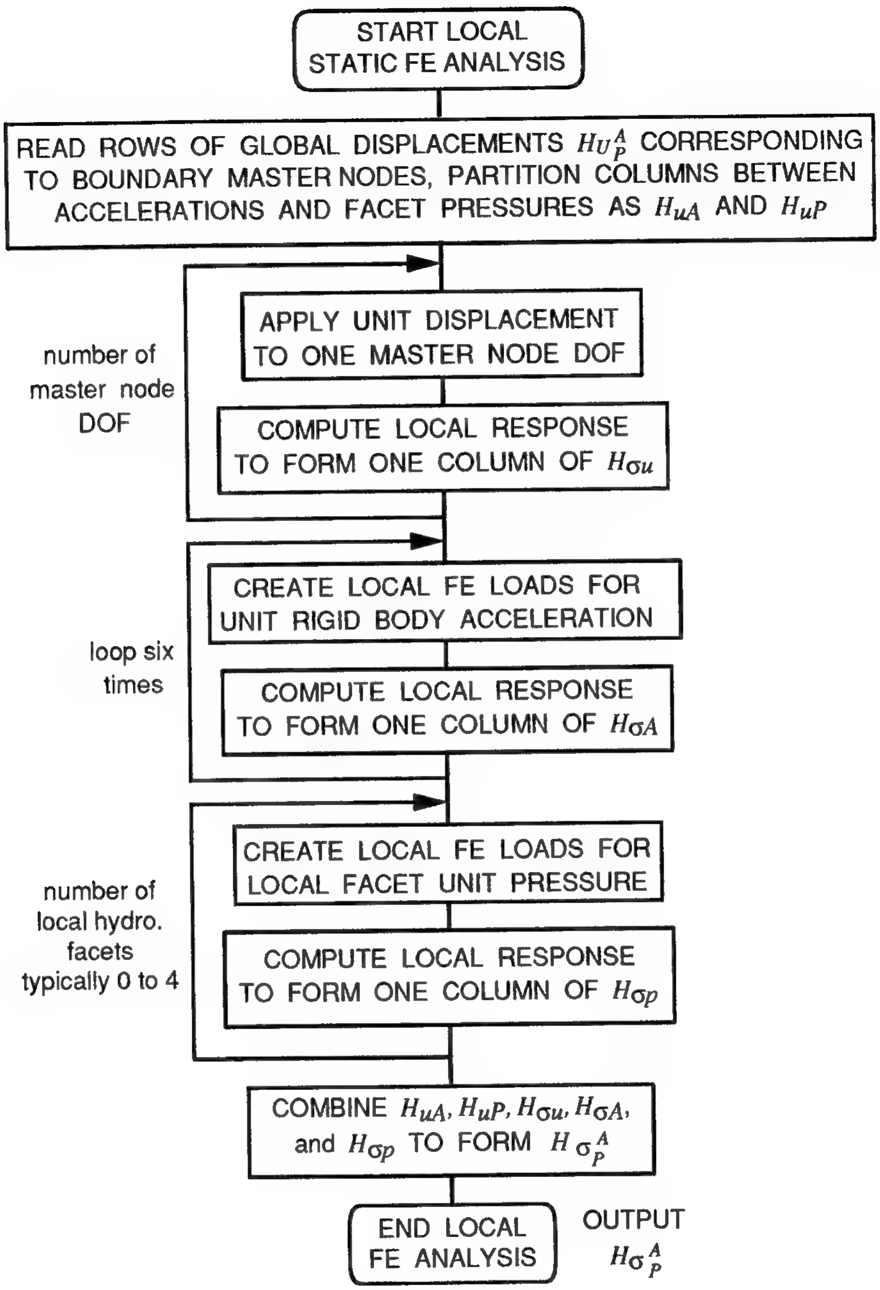
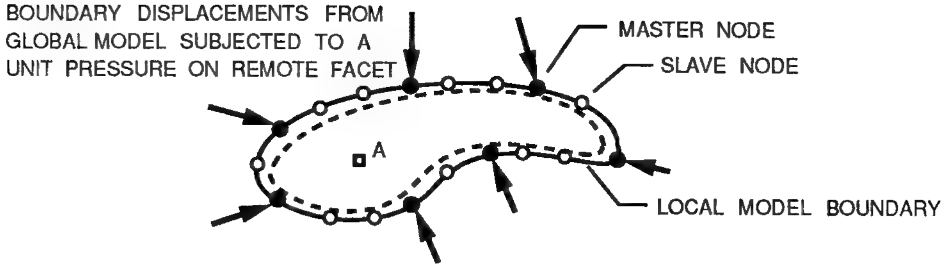
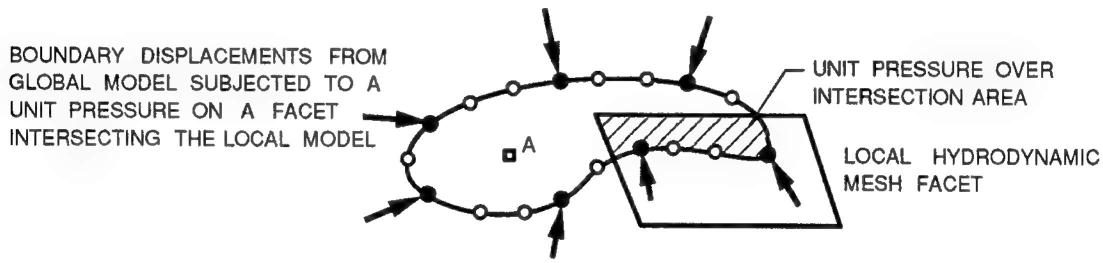


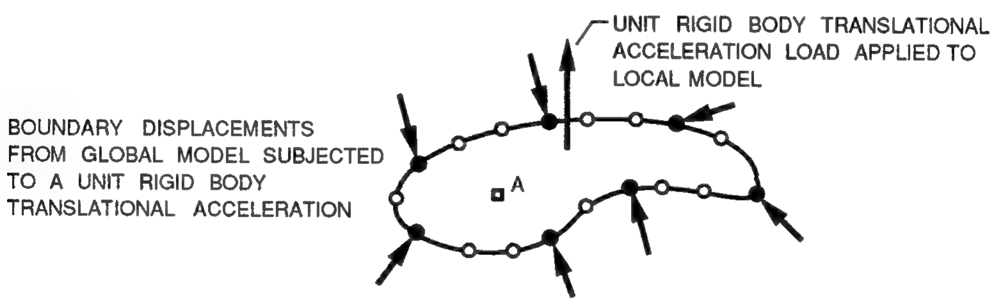

Figure 7: Flow chart for the Local Unit Load Method of calculating the transfer function relating the hydrodynamic facet pressures and rigid body accelerations to the local response



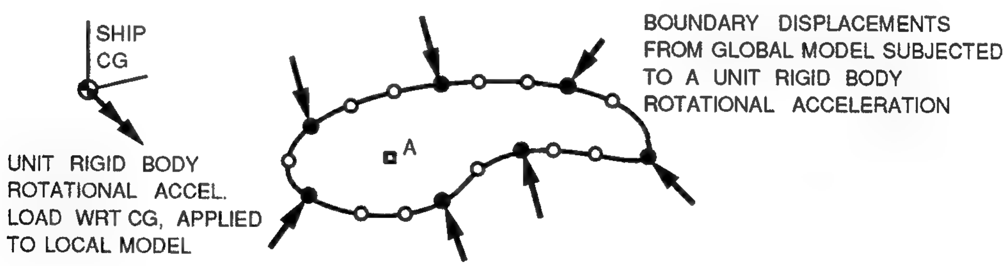

a) Displacements applied at all master node DOF (from global analysis with unit pressure on a hydrodynamic facet remote from local model)




b) Displacements applied at all master node DOF (from global analysis with unit pressure on hydrodynamic facet intersecting the local model) plus local facet unit pressure




c) Boundary displacements (from global analysis with unit surge, sway, or heave acceleration loads) plus associated local acceleration loads




d) Boundary displacements (from global analysis with unit roll, pitch, and yaw acceleration loads) plus local acceleration loads due to associated accelerations about ship CG

Figure 8: Example load cases for quasi-static local analyses with the Unit Facet Pressure Method to determine structural response at internal node $A$

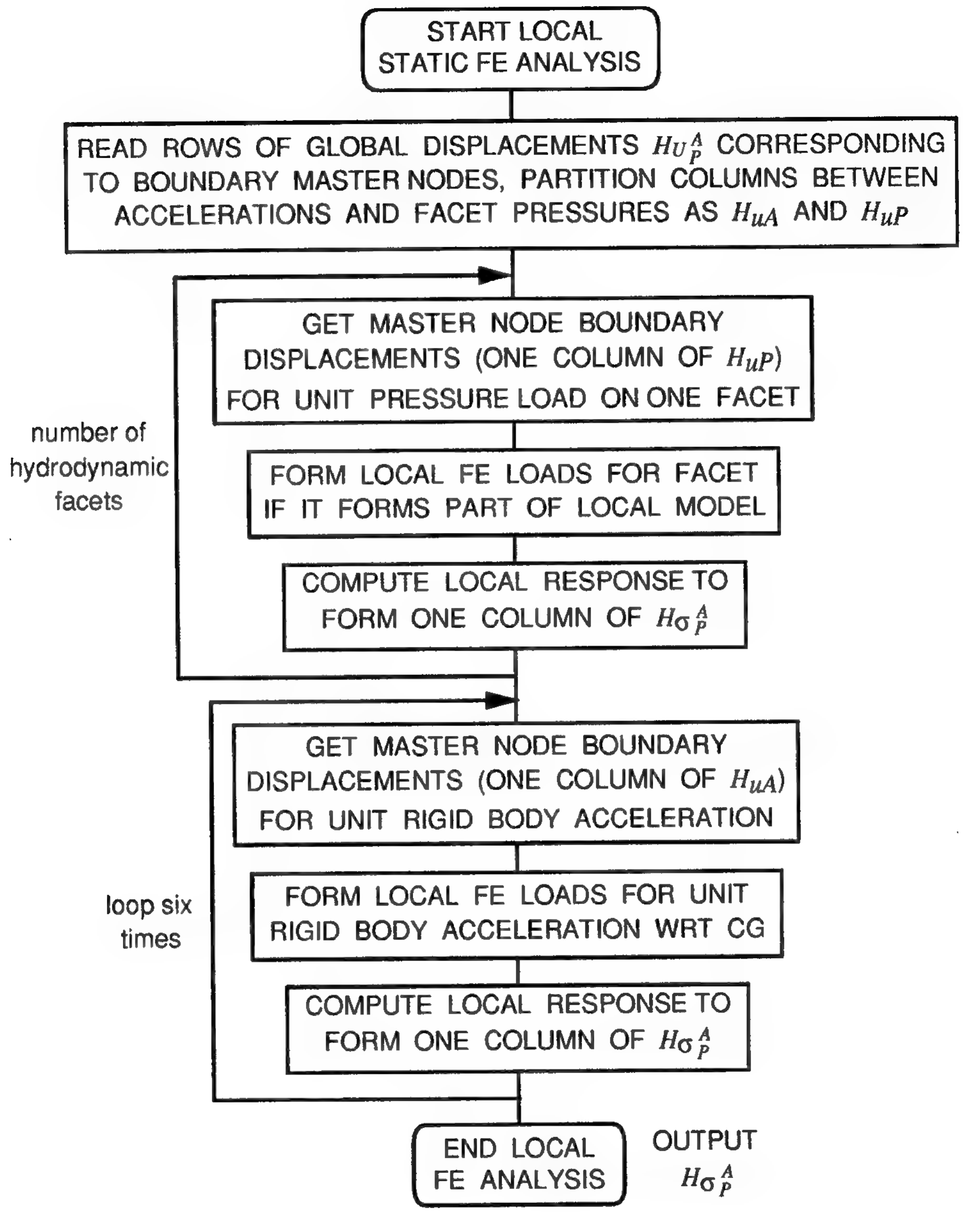



Figure 9: Flow chart for the Unit Facet Pressure Method of calculating the transfer function relating the hydrodynamic facet pressures and rigid body accelerations to the local response

ship speed $V_k$) needed to represent the cells defining a ship operational profile. The number of PRECAL load cases $n_l$ is given by

$$n_l = n_\omega \times n_\theta \times n_V, \tag{30}$$

the product of the number of discrete wave frequencies, heading angles and ship speeds needed to represent the operational profile. The direct method requires $2n_l$ load cases. The local unit load method requires $n_u + n_p + 6$ load cases where $n_u$ is the number of local model boundary master node DOF and $n_p$ is the number of hydrodynamic facets intersecting the local model (likely 0 to 4). The unit facet pressure method requires $n_P + 6$ load cases where $n_P$ is the number of the facets used in the PRECAL hydrodynamic mesh.

If the ship operational profile is very simple, defined for example, in terms of a single operational cell for long crested waves (one heading angle, one ship speed and perhaps twenty discrete wave frequencies), then the direct method would require 40 load cases. A realistic short term or long term operation could involve perhaps 8 headings, 20 frequencies and 5 ship speeds resulting in 1600 load cases. Even if operational cells are defined in terms of 5 headings (head, bow, beam, quartering, following seas) dividing these headings between port and starboard would result in 8 heading angles. It may also be desirable to consider a larger number of headings in the PRECAL transfer functions (perhaps at 15 degree increments giving 24 headings) since the pressure and acceleration transfer functions can exhibit strong variation with heading angle in some cases. The ship operational information could still be specified for 45 degree sectors but the time spent in these sectors would be split between the smaller angular intervals used in the PRECAL transfer functions. If analyses are based on wave buoy data, a desirable option for experimental validation purposes, wave spectra could be specified at 0.01 Hz increments (0.03 Hz to 0.3 Hz) and 10 degree heading angle increments, resulting in 2000 load cases with the direct method for each ship speed. The direct method is probably closest to the methods presently implemented in VAST, but under realistic operational profiles it is likely to require considerably more load cases than the other two methods proposed as will be shown below.

Based on a typical hydrodynamic mesh containing perhaps 200 facets, the unit facet pressure method would require 206 local load cases. If the global finite element model is relatively coarse then the local model could contain a relatively small number of boundary master nodes, perhaps 20 to 40, giving 120 to 240 DOF which would result in 130 to 250 load cases if the local unit load method is employed. This would indicate that the unit local load method and the unit facet pressure method would require roughly the same number of local analyses. The unit facet pressure method calculates the transfer function $\left[H_{\sigma^A_P}\right]$ directly while the local unit load method forms this transfer function from the addition and multiplication of five other transfer functions (refer to Equation 28). Thus the local unit load method would require additional multiplication and addition of transfer functions but since the largest matrices involved are relatively small (perhaps 250 x 250 elements) this difference may not be significant compared to the actual computation time required for the local finite element analysis. Since the unit facet pressure method is an extension of the global analysis method and will use similar algorithms for applying facet pressures it is likely to require less effort to implement. In the detailed computer implementation of either method further advantages or disadvantages may be discovered and in the end it will likely be desirable to implement both the local unit load and unit facet pressure methods.

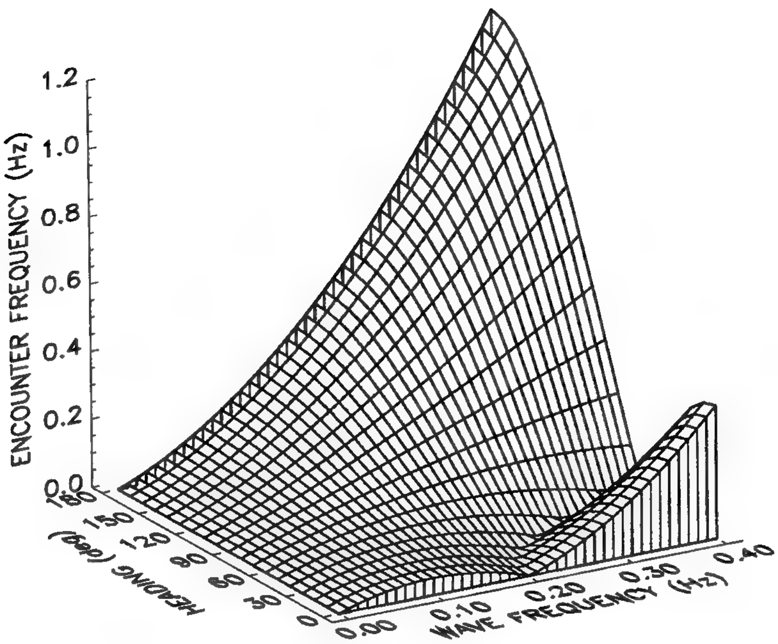



Figure 10: Plot of wave encounter frequency $\omega_e$ as a function of the wave frequency $\omega$ and heading angle $\theta$ for a ship speed of 15 knots

# 5 Encounter Frequency Spectra

For use in fatigue and crack growth analysis it is desirable to consider the local response in terms of encounter frequency $\omega_e$ (the actual frequency seen by the structure) based on the response spectra $[S_{\sigma\sigma}(\omega_e)]$ for each operational cell. The encounter frequency is given by

$$\omega_e = |\omega - \omega^2 V \cos(\theta)/g| \tag{31}$$

where $g$ is the acceleration of gravity and $V$ is the ship speed. The encounter frequency, a quadratic function of the wave frequency, is plotted in Figure 10 over a range of wave frequencies and heading angles between 0 degrees (following sea) and 180 degrees (head sea) for a ship speed of 15 knots. The plot exhibits a 'valley' along the line of zero encounter frequency.

For an operational cell with ship speed $V_k$, and wave directional spectral density $S_{\eta\eta}(\omega, \theta)$, defined over discrete frequencies $\omega_i$, $i = 1, n_\omega$ and heading angles $\theta_j$, $j = 1, n_\theta$ as $S_{\eta\eta}^{ij}$, Equation 17 can be used to calculate the local response $[S_{\sigma\sigma}(\omega, \theta)]$ again defined discretely as $[S_{\sigma\sigma}]_{ij}$. $[S_{\sigma\sigma}(\omega_e)]$ can be obtained by integrating the response spectral energy density $S_{\sigma\sigma}(\omega, \theta)$ between lines of constant encounter frequency with spacing $\delta_{\omega_e}$. Figure 11 shows plots of a hypothetical spectrum in terms of contours of constant $S_{\sigma\sigma}$ overlaid with lines of constant encounter frequency 0.01 Hz apart centered at 0.05, 0.10, 0.20 and 0.30 Hz. Integration of the spectral energy density in the shaded area between each set of lines would give the spectral energy in a

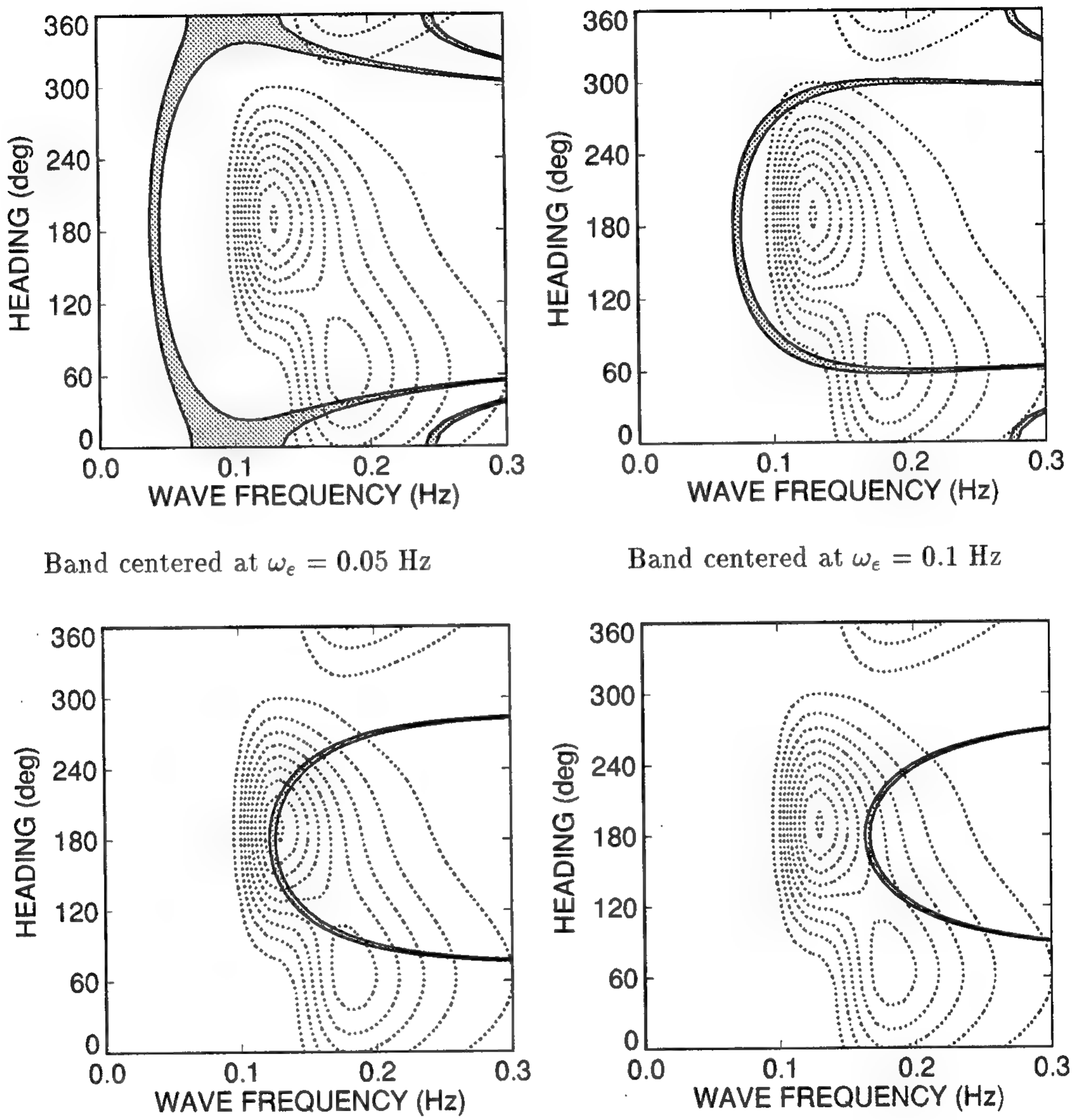
bandwidth of 0.01 Hz about the specified center frequencies. The figure shows that wave energy over a broad range of wave frequencies can contribute to the response energy in a narrow band of encounter frequency.

The energy spectra $[S_{\sigma\sigma}(\omega_e)]$ can be obtained numerically by simply calculating terms $[S_{\sigma\sigma}]_{ij}\,\delta_\omega\delta_\theta$ where $\delta_\omega$ and $\delta_\theta$ are the wave frequency and heading angle point spacing. This response energy is added to the appropriate frequency bin based on the encounter frequency $\omega_e^{ij}(\omega_i, \theta_j)$. For experimental wave directional spectra this approach tended to produce 'jagged' spectra [4]. Smooth spectra were obtained if bilinear interpolation of the $[S_{\sigma\sigma}]_{ij}$ points was used to make $\delta_\omega$ and $\delta_\theta$ smaller. Reducing the angular and frequency spacing to $\delta_\omega/4$ and $\delta_\theta/4$ and maintaining bins of encounter frequency of width $\delta_{\omega_e}$ equal to the original wave frequency spacing $\delta_\omega$ produced smooth spectra. It is possible to employ higher order integration schemes but it was found that this did not work well for measured directional spectra which tended to have sharp spikes with high energy at some points and low energy at surrounding points.

If the cell wave spectra are unidirectional then obtaining $[S_{\sigma\sigma}(\omega_e)]$ is simpler. This may be the case in ISSMM where a cell is planned to be defined in terms of the Bretschneider two-parameter model for long crested waves with spectral density $S_{\eta\eta}(\omega)$ defined by the modal period and significant wave height. In this case, for a given ship speed $V_k$, heading angle $\theta_j$, and wave spectral density $S^i_{\eta\eta}$ defined at discrete wave frequencies $\omega_i$, $i = 1, n_\omega$, the response spectral density is given by

$$[S_{\sigma\sigma}]_i = \{H_{\sigma\eta}\}_{ijk}\, S^i_{\eta\eta} \left\{H^*_{\sigma\eta}\right\}^T_{ijk}, i = 1, n_\omega \qquad (32)$$

While it is simpler to convert $[S_{\sigma\sigma}(\omega)]$ to $[S_{\sigma\sigma}(\omega_e)]$ than converting $[S_{\sigma\sigma}(\omega,\theta)]$ to $[S_{\sigma\sigma}(\omega_e)]$ it is not trivial since at some headings three values of wave frequency can contribute energy to the same encounter frequency. A numerical method similar to that proposed for treating $[S_{\sigma\sigma}(\omega,\theta)]$, but reduced to one dimension, could be employed. An overall flow chart summarizing the global and local analyses is shown in Figure 12 for the case where the operational cell is defined for long-crested seas. A loop is shown for re-meshing the local model, after an increment of crack growth for the case of a crack propagation analysis, although in this case as previously mentioned, the local response spectrum would be based on stress intensity factor $K$ and not the stress $\sigma$ shown in the figure.




Band centered at $\omega_e = 0.05$ Hz

Band centered at $\omega_e = 0.1$ Hz

Band centered at $\omega_e = 0.2$ Hz

Band centered at $\omega_e = 0.3$ Hz

Figure 11: Plots of directional spectra $S_{\sigma\sigma}(\omega, \theta)$ overlaid with 0.01 Hz wide bands of encounter frequency; $\cdots$ contours of response spectral density, — bounds of the encounter frequency band

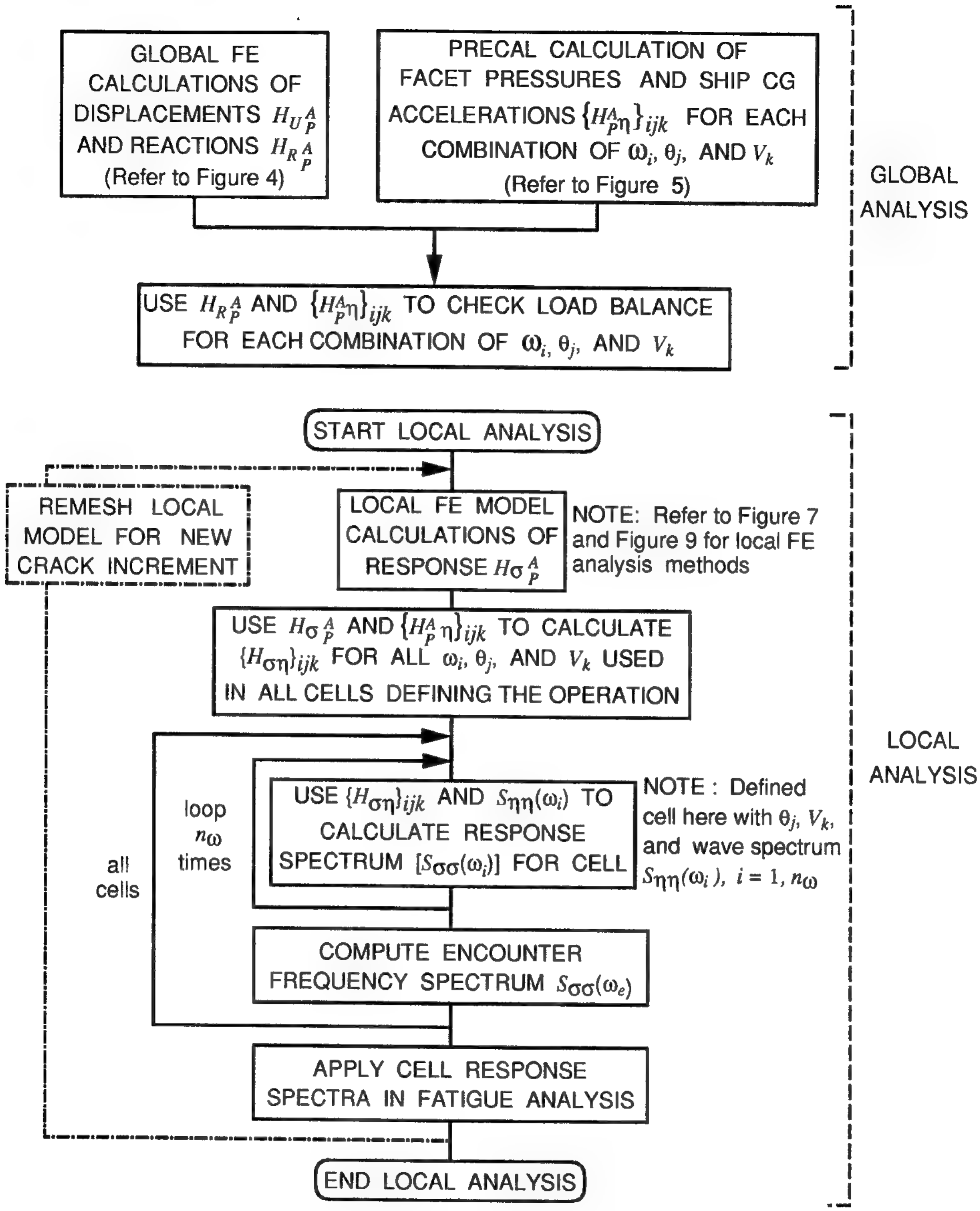



Figure 12: Overall flow chart for global and local analyses for the case where an operational cell is defined for a single heading angle $\theta$, ship speed $V$, and a unidirectional wave spectrum $S_{\eta\eta}(\omega)$

# 6 Summary and Conclusions

This report proposes a top-down finite element procedure for the prediction of structural response frequency spectra from wave spectra and regular wave pressure loads predicted with a 3D linear hydrodynamics code. These spectra can be used for assessment of fatigue strength and possibly ultimate strength based on realistic ship operational profiles.

The method employs a quasi-static approach based on finite element computations for unit pressures applied to hydrodynamic mesh facets. Transfer functions between regular waves and the structural response are employed, eliminating the need to calculate huge cross spectral density matrices of hull pressure loads as used in classical random response methods. Separate global static finite element analyses are conducted with a unit pressure on each facet of the hydrodynamic mesh and for six rigid body acceleration load cases. The resultant global model displacements can be stored and then used for any subsequent local model analyses with any number of operational cells, employing different wave spectra and ship speeds and headings, without having to repeat the global finite element analysis.

The quasi-static approach requires constraint of the global model against rigid body motion. The method also leads to a relatively efficient procedure for checking reaction forces and inertial loads for any of the PRECAL regular wave load cases employed in the analyses.

Two methods are recommended for the local detailed model analyses. In the first, called the local unit load approach, separate static finite element analyses are conducted for load cases of a master node unit displacement, unit rigid body accelerations, and unit local hydrodynamic facet pressures loads. In the second, called the unit facet pressure method, separate finite element static analyses are conducted based on simultaneous application of all boundary node displacements and associated local pressure or acceleration loads resulting from each global model unit facet pressure or unit rigid body acceleration load case. The relative efficiency of the methods will depend on the number of master nodes on the boundary of the local model compared to the number of hydrodynamic mesh facets. Implementation and testing of both methods is recommended before the final choice between methods is made.

It is anticipated that the proposed approach should make spectral methods, based on hull pressure wave loads, sufficiently efficient to be practically applied within ISSMM. The approach is to be implemented in the VAST suite of codes and tested for application to fatigue analysis as part of the first phase of ISSMM. If the method is found to give realistic predictions with an acceptable computational effort, it will be refined and integrated into the ISSMM software. An investigation of the use of the method to handle nonlinear sea loads will also be undertaken.

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Ship structural design and analysis is moving away from empirical static design wave balance and towards more rational methods involving computer modeling of the sea load and structural response. A ship load history is usually only known or predicted in terms of an operational profile defined by the time spent in many combinations of different ship speeds, headings and sea environments. Each combination of speed, heading the wave frequency spectrum defining the sea environment can be used to derive a frequency spectrum of the local structural response (such as stress, strain or stress intensity factor) at a location or region of the ship. The resulting response spectra can then be applied in a fatigue or ultimate strength assessment. This report proposes a method for the calculation of the frequency spectrum of the structural response based on the use of regular wave hull pressure loads and rigid body accelerations provided by PRECAL (a linear frequency domain hydrodynamics code based on 3D potential flow) and a top-down quasi-static structural analysis procedure to be implemented in the DND suite of finite element codes called VAST. Static finite element analyses are conducted for unit hydrodynamic facet pressure and rigid body acceleration load cases. This should reduce tremendously the computational effort required compared to directly applying a set of wave pressure loads for each combination of regular wave frequency, ship speed and heading needed to represent a ship operational profile. Transfer functions relating a regular unit amplitude wave directly to the structural response are calculated before computing the response spectra, eliminating the need to explicitly apply large cross spectral density matrices of hull pressure loads to the finite element model as is often done in classical random response methods.

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ship structural analysis
finite element
ocean wave spectra
random response
hull pressure